



# Coupled Electron and Nuclear Spin Dynamics in InAs Quantum Dots: Impact on Single and Two-Qubit Operations

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## Approach

- Atomistic model for hyperfine interaction constant
- Classical dynamics of coupled electron and nuclear spins
- Nuclear spin-spin interaction neglected
- Compute effect on electron spin flip operation

## Results

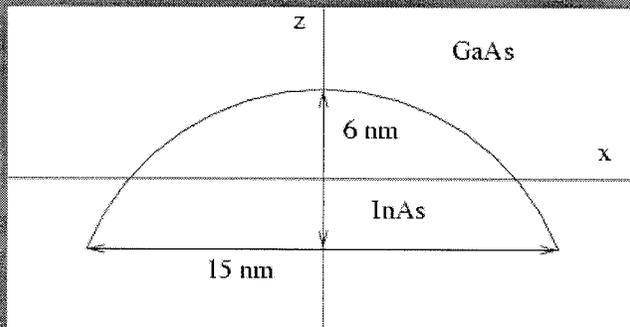
- Nuclear magnetic field varies on the time scale of QC
- The induced error is of the order of  $1e-2$

Funding from ARDA and JPL Research and Technology Development



# Electron-Nuclear Spin Interaction

## Quantum dot



## Hyperfine constant

$$A_j = \frac{16\pi}{3} \mu_B \mu_j |\Psi(R_j)|^2$$

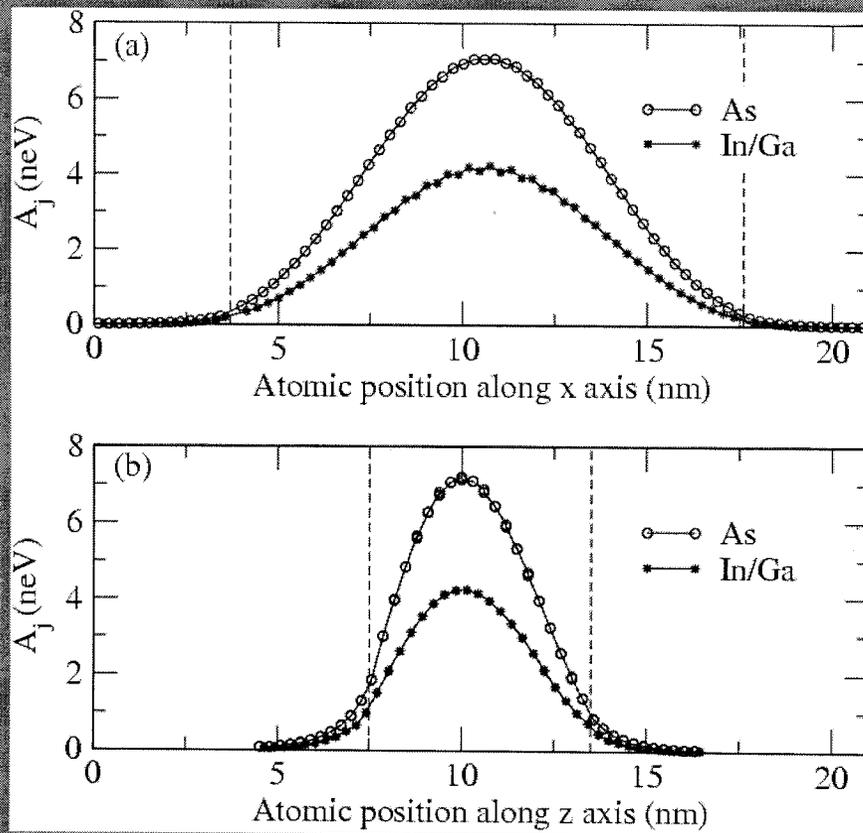
Typical  
light barrier

## Hyperfine interaction Hamiltonian

$$H_{HF} = \sum_j A_j I_j \cdot S \equiv g_e \mu_B B_N \cdot S$$

## Effective Nuclear Magnetic Field

$$B_N = \frac{1}{g_e \mu_B} \sum_j A_j \langle I_j \rangle$$





# Coupled electron and nuclear spin dynamics

## Equations of motion

$$\partial_t \langle S_\alpha \rangle = -\varepsilon_{\alpha\beta\gamma} \left( \sum_{j=1}^N A_j \langle S_\beta I_{j\gamma} \rangle + g_{el} \mu_B \langle S_\beta \rangle (B_0 + B_{ac}(t))_\gamma \right) \cong -g_{el} \mu_B \varepsilon_{\alpha\beta\gamma} \langle S_\beta \rangle (B_N + B_0 + B_{ac}(t))_\gamma$$

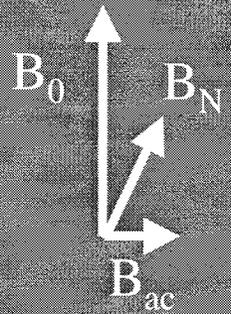
$$\partial_t \langle I_{j\alpha} \rangle = -\varepsilon_{\alpha\beta\gamma} \left( A_j \langle I_{j\beta} S_\gamma \rangle + g_n \mu_n \langle I_{j\beta} \rangle (B_0 + B_{ac}(t))_\gamma \right) \cong -g_n \mu_n \varepsilon_{\alpha\beta\gamma} \langle I_{j\beta} \rangle (B_j + B_0 + B_{ac}(t))_\gamma$$

$$B_{j\gamma} = \frac{A_j \langle S_\gamma \rangle}{g_n \mu_n}, \quad B_{N\gamma} = \frac{1}{g_{el} \mu_B} \sum_{j=1}^N A_j \langle I_{j\gamma} \rangle$$

- Nuclear spin-spin interaction neglected
- No electron-nuclear spin state entanglement
- Dynamics is “faster” for electron than for nuclear spins
- Amplitude of  $B_N$  is not constant in III/V materials



# Single Qubit Operation with ESR



**Electron Spin Precession Frequency**

$$\omega_e = g_e \mu_B \sqrt{(B_0 + B_N^{\parallel})^2 + (B_N^{\perp})^2}$$

**ESR Tuning Condition**

$$B_{ac}(t) = B_{ac} (e_x \cos \omega_{ac} t + e_y \sin \omega_{ac} t)$$

$$\omega_e = \omega_{ac}$$

If  $B_N$  changes in time after gate calibration, the ESR field is detuned!

$$\omega_e \neq \omega_{ac}$$

**Rabi formula**

$$P_{\uparrow\downarrow} = \frac{\omega_B^2}{\omega_B^2 + (\omega_{ac} - \omega_e)^2} \sin^2 \left( \sqrt{\omega_B^2 + (\omega_{ac} - \omega_e)^2} \frac{t}{2} \right)$$

**Spin Hamiltonian**

$$H = \frac{\hbar}{2} \begin{pmatrix} \omega_e & \omega_B e^{-i\omega_{ac}t} \\ \omega_B e^{i\omega_{ac}t} & -\omega_e \end{pmatrix},$$

where  $\omega_B = g_e \mu_B B_{ac}$

$\pi$  flip

$$t_{flip} = \frac{\pi}{\sqrt{\omega_B^2 + (\omega_{ac} - \omega_e)^2}}$$

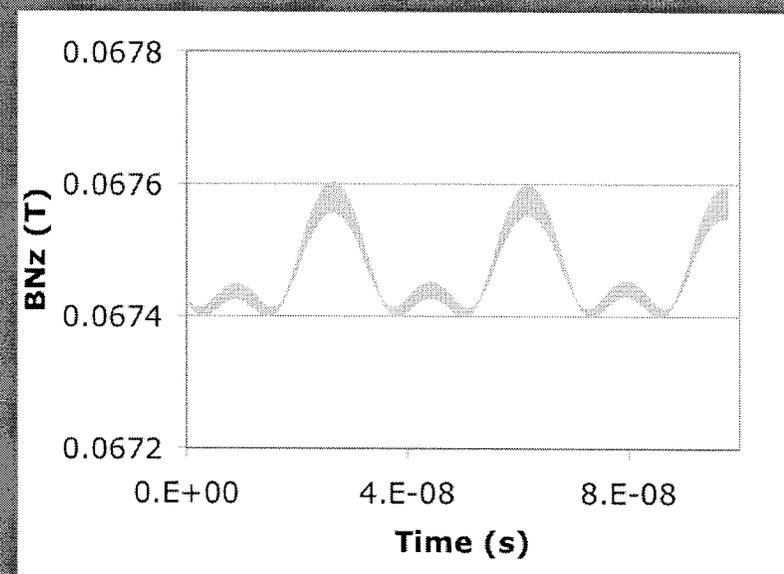
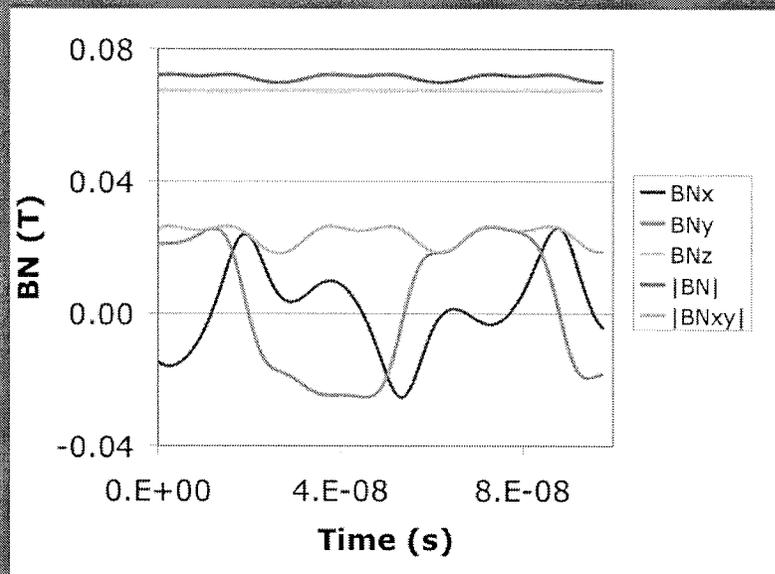
**Flip Error:**

$$\left( \frac{\omega_{ac} - \omega_e}{\omega_B} \right)^2$$



# Nuclear spin dynamics

- Number of atoms is about 20,000
- $B_0=1$  T,  $B_{ac}=1e-3$  T
- ESR resonance at  $t=0$

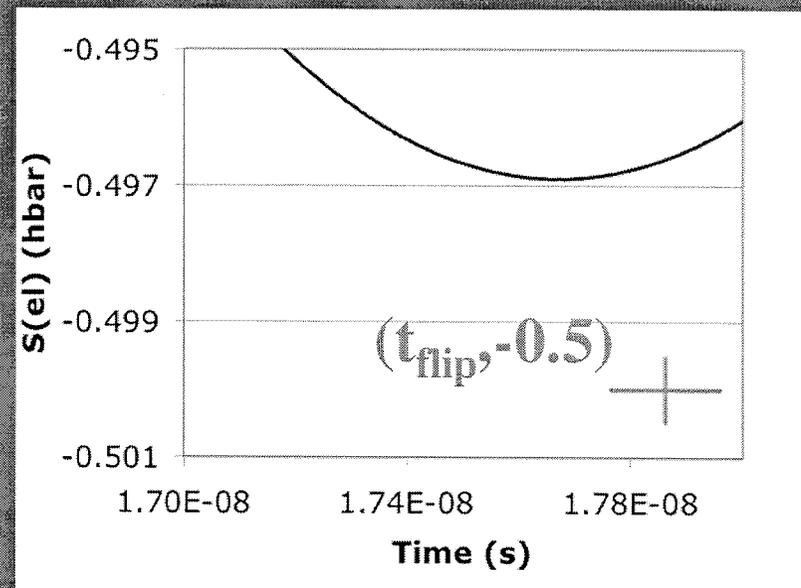
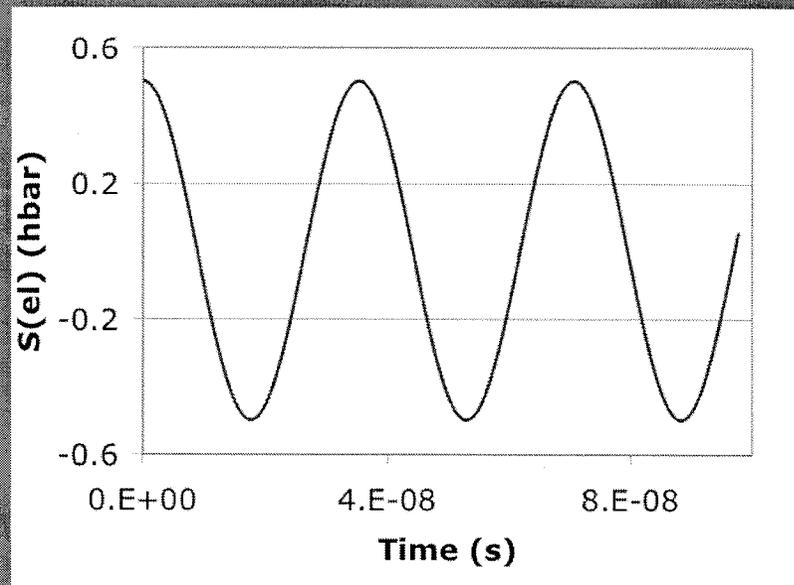


- Amplitude of  $B_N$  is not constant
- $B_{Nz}$  fluctuates by about  $2e-4$  T
- Maximum expected error on spin flip operation is  $4e-2$



# Electron spin dynamics

- Number of atoms is about 20,000
- $B_0=1$  T,  $B_{ac}=1e-3$  T
- ESR resonance at  $t=0$
- At ESR resonance  $t_{flip}=1.786e-8$  s



**Conclusion:** Nuclear spins need to be polarized to meet the quantum computing error correction threshold of  $1e-4$ .



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